

Name: _____ Student ID: _____ Date: _____

Test #2 - 201-316-VA - Fall 2020

ALWAYS justify your answers. Do not offer or accept any outside assistance:

(1) (2 points) Consider a sample space S containing events A, B , and C :

(a) Suppose that...

$$P(A) = 0.45,$$

$$P(B) = 0.52,$$

$$P(C) = 0.13,$$

$$P(A \cap B) = 0.66,$$

$$P(A \cup B) = 0.3,$$

$$\text{and } P(C') = 0.7.$$

Explain everything that is wrong with the above statements:

(b) Suppose A, B , and C are exhaustive and mutually-exclusive, and suppose instead that...

$$P(A) = 0.1,$$

$$P(B) = 0.2,$$

$$P(C) = 0.4,$$

$$P(B \cap C) = 0.05,$$

$$P(A \cup C) = -1,$$

Explain everything that is wrong with the above statements:

- (2) (3 points) Ideologically-mixed marriages occur when spouses have opposite political beliefs. According to a 2016 study, men in these marriages are 66% likely to be more conservative, while women in these marriages are 66% likely to be more liberal. Let M , W , C , and L be the events “is a man”, “is a woman”, “is conservative”, and “is liberal” respectively:

(a) Explain the difference between the event that a randomly-selected person “is conservative *and* is a man”, and “is conservative *given that* they’re a man”:

(b) Suppose in a sample of 100 ideologically-mixed married couples that there are exactly 100 men and 100 women. Find the probability that a randomly-selected person (i) is both conservative *and* a man, and (ii) is conservative *given that* they’re a man:

(c) Find the probability that a randomly-selected person in this sample is a man, *given that* they’re conservative:

(d) Under what conditions, in general, is it true that $P(A|B) = P(B|A)$?

- (3) **(3 points)** (a) Determine which of the following could be described as either permutations, combinations, or neither, and find the total number for each:
- (i) All the different possible lotto 6/49 tickets:
 - (ii) All the possible outcomes of rolling two six-sided dice:
 - (iii) All the different ways a gold medal, a silver medal, and a bronze medal can be awarded to a group of ten people:

(b) Find the number of different ways you could reorder the digits in your student ID:

(c) Suppose six students are randomly assigned seats at the front row of a gym in which an exam is being written. Suppose further that one of the students wishes to sit next to the window. How many seating arrangements result in them sitting next to the window?

(d) Suppose they want to sit right next to their friend, who is also sitting in the same row, and now they don't care about sitting next to the window. How many seating arrangements result in them sitting next to each other?

(4) **(3 points)** Suppose a secret agent has only one attempt to disarm a nuclear bomb with a 5-digit deactivation code:

(a) What is the probability they successfully defuse the bomb if digits *can* be repeated?

(b) What is the probability that they successfully defuse the bomb if digits are *not* allowed to be repeated, and they can be entered in *any order*? (ie, 12345 would be considered the same code as 34512)

(c) What is the probability they successfully defuse the bomb if digits are *not* allowed to be repeated, but they have to be entered in a specific order?

(d) Which of these scenarios is safest?

- (5) (3 points) Suppose 5% of Canadians are considered wealthy, and the probability of a person being educated *given that* they are wealthy is 0.80. Suppose further that the probability of a person being educated *given that* they are *not* wealthy is 0.30:

(a) What is the probability of a Canadian being wealthy *and* educated?

(b) Explain, logically, why the probability of a person being **educated** is equal to the probability of being **educated and wealthy**, plus the probability of being **educated and not wealthy**:

(c) What is the probability of a person being wealthy, *given that* they are educated?

- (6) **(3 points)** Consider a game at a carnival where two six-sided dice are rolled, and players wager money trying to predict whether the result is over 7 or under 7:

(a) Suppose the last five totals were 12, 8, 8, 11, 10, and two different players at the carnival react differently to this information: one of them, Brad, says, “this means numbers exceeding 7 are more likely” and bets that the next result will exceed 7. The other player, Chad, says, “actually, a number below 7 is due to happen any moment now!”, and bets that the next result will be less than 7. Explain in detail which of them you think is right:

(b) Suppose a player wagers \$10 and, if they correctly predict whether the next two dice will sum to a value greater than 7, they earn \$8 more (in addition to the \$10 they wagered); however, if the dice values are less than 7, they lose their wager and earn nothing. If the dice land on exactly 7, nothing happens, and they get their wager back. Calculate the expected value of playing this game; would you bet money on this game?

- (7) (3 points) Suppose a person is taking a multiple-choice quiz in a subject they have absolutely no knowledge of, so they randomly guess the answer to each question. Suppose further that there are eight questions, and each question has four possible choices. Let x be the number of correct answers they get on the quiz. Then x is a binomial random variable in an experiment where the two possible outcomes for each question are getting a right answer, or getting a wrong answer:

(a) What is the “probability of success” p in this experiment, and what is the “number of trials” n ?

(b) What is the expected number of correct answers?

(c) What is the probability of getting *exactly* one correct answer?

(d) What is the probability of getting none of the right answers?

(e) What is the probability of getting *more than* one correct answer?